INFINITY THINK BEYOND.....
AN EDUCATIONAL INSTITUTE

Concept Sheet

Real numbers

Types of Numbers

Natural Numbers:
N = {1,2,3,4,5.....}
It is the counting numbers

Whole number:
W= {0,1,2,3,4,5.....}
It is the counting numbers + zero

Integers:
Z={...-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6...}

Positive integers:
Z+= {1,2,3,4,5.....}

Negative integers:
Z-={...-7,-6,-5,-4,-3,-2,-1}

Prime numbers :
A number is called a prime number, if it has no factor other than 1 and the number itself.
e.g., 3, 11, 23, 43, 71 are prime numbers.

Composite numbers :
A number is called a composite number, if it has atleast one factor other than 1 and the number itself.
e.g., 4, 6, 24, 45, 188 are composite numbers.

KEY POINTS

(i) It should be noted that the 1 is neither prime nor composite.
(ii) 2 is the smallest prime number. It is the only even prime number. All other even numbers are composite numbers.
(iii) If a number is not divisible by any one of the primes less than half of it, then it is prime, otherwise it is composite.
(iv) Prime numbers have only two factors, 1 and the number itself. As 1 is not a prime number, do not include it when expressing a number as a product of prime factors.

FUNDAMENTAL TEOREM OF ARTHIMETIC

Every composite number can be written as the product of primes and this factorisation is unique, apart from the order in which the prime factors occur. Fundamental theorem of arithmetic, is also called unique factorisation theorem. Composite number = Product of prime numbers.

Or

Any integer greater than 1 either be a prime number or can be written as a unique product of prime numbers.
To find the HCF and LCM by prime factorisation method, firstly express the given two or more numbers into the product of prime numbers separately. Then, HCF of two or more numbers = Product of the smallest power of each common prime factor involved in the numbers.
LCM of two or more numbers = Product of the greatest power of each prime factor involved in the numbers. For any two positive integers a and b, we have ,

$$HCF\ (a, b) \times LCM\ (a, b) = a \times b$$

$$\therefore HCF\ (a, b) = \frac{a \times b}{LCM\ (a,b)} \quad \text{or} \quad LCM\ (a,b) = \frac{a \times b}{HCF\ (a,b)}$$

PRACTICE SHEET 1

Q1.Express each of the following integers as a product of its prime factors :

(i) 420 (ii) 468
(iii) 945 (iv) 7325

Q2.Determine the prime factorisation of each of the following positive integer :

(i) 20570 (ii) 58500 (iii) 45470971

Q3.Find the LCM of 135 and 225 by the prime factorization method.

Q4.Find the LCM of 196 and 38220 by the prime factorization method.

Q5.Find the HCF and LCM of 90 and 144 by the prime factorisation method.

Q6.Find the HCF of 867 and 255 by the prime factorization method.

Q7.Find the HCF and LCM of 32 and 54 by the prime factorization method.

Q8.Find the HCF and LCM of 18 and 24 by the prime factorization method.

Q9.Find the HCF and LCM of 144, 180 and 192 by prime factorisation method.

Q10.Find the HCF and LCM of 510 and 92 by prime factorisation method.

Q11. The LCM and HCF of two numbers are 180 and 6 respectively. If one of the numbers is 30, find the other number.

Q12.The HCF of two numbers is 16 and their product is 3072. Find their LCM.

Q13.The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.

Q14.Given that HCF (306, 1314) = 18, find LCM (306, 1314).

Q15.Find the LCM and HCF of 120, 105 and 150 by the prime factorization method.

Q16.Given that HCF (306, 657) = 9, find LCM (306, 657).

PRACTICE SHEET 2

Q17.Find the HCF and LCM of the following integers by prime factorization method :

(i) 24, 60, 150 (ii) 6, 72, 120

Q18.Given that HCF (2520, 6800) = 40 and LCM (2520, 6800) = 252 K . Find the value of K.

Q19.If HCF(12576, 48624) = 12 K, find the value of K.

Q20. Find the LCM and HCF of the following pairs of integers and verify that

LCM x HCF = Product of the integers :

(i) 26 and 91 (ii) 510 and 92

Q21.For positive integers a and b, write the relation between a, b, HCF (a, b) and

LCM (a, b).

Q22.Show that $2 \times 3 \times 7 \times 13 \times 17 + 13$ is a composite number.

Q23.Show that 23 is a prime factor of the number $2 \times 7 \times 11 \times 17 \times 23 + 23$.

Q24. Show that the number 7^n can not end with the digit 0.

Q25. Show that the number 5^n can not end with the digit 0.

Q26.Show that the number 9^n can not end with the digit 2 for any natural number n.

Q27.Show that the number 5^n can not end with the digit 2 for any natural number n.

Q28.Show that the number 13^n can not end with the digit 0.

Q29.Explain why 7 x 11 x 13 + 13 and 7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 are composite number .

Q30.Show that $9 \times 7 \times 6 \times 5 + 6 \times 5$ is a composite number.

PRACTICE SHEET 3

Q31. Prove that $\sqrt{3}$ is an irrational number.

Q32. Prove that $\sqrt{11}$ is an irrational number.

Q33. If p is a prime number, then prove that $\sqrt{p}$ is an irrational number.

Q34. Show that $2\sqrt{3}$ is an irrational number.

Q35. If a is a nonzero rational and $\sqrt{a}$ is irrational, then show that $a\sqrt{b}$ is an irrational number.

Q36. Show that $(4 - \sqrt{3})$ is an irrational number.

Q37. If a is rational and $\sqrt{b}$ is irrational, then prove that $(a + \sqrt{b})$ is an irrational number.

Q38. Show that $(5- 2\sqrt{3})$ is an irrational number.

Q39. Show that $\frac{1}{\sqrt{2}}$ is an irrational number.

Q40. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Q41. Prove that $(\sqrt{2} + \sqrt{3})$ is an irrational number.

Q42. Classify the $\frac{22}{7}$ as rational or irrational.

Q43. Classify the 3.1416 as rational or irrational.

Q44. Classify the $3.\overline{142857}$ as rational or irrational.

Q45. Classify the 5.636363 as rational or irrational.

Q46. Prove that $\sqrt{6}$ is an irrational number.

Q47. Prove that $(2 - \sqrt{3})$ is an irrational number.

Q48. Prove that $(5 + 3\sqrt{2})$ is an irrational number.

Q49. Prove that $3\sqrt{7}$ is an irrational number.

Q50. Prove that $\frac{3}{\sqrt{5}}$ is an irrational number.

Q51. Prove that $(2 - 3\sqrt{5})$ is an irrational number.

Q52. Prove that $(\sqrt{3} + \sqrt{5})$ is an irrational number.

Q.53. Prove that $\frac{1}{\sqrt{3}}$ is an irrational number.

Q.54. Give an example of two irrational number whose sum is a rational number .

Q.55. Give an example of two irrational number whose product is a rational number .

PRACTICE SHEET 4 (ADDITIONAL QUESTIONS)

Q.56. Find the largest number which divides 245 and 1037, leaving remainder 5 in each case.

Q.57. Find the least number which when divided by 35, 56 and 91 leaves the same remainder 7 in each case.

Q.58. Find the smallest number which when divided by 28 and 32 leaves remainders 8 and 12 respectively.

Q.59.Find the greatest number of four digits which is exactly divisible by 15, 24 and 36.

Q.60. Find the least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3

Q.61. Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm.

Q.62. If HCF of 65 and 117 is expressible in the form 65n – 117,then find the value of n.

Q.63. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is

600. If one number is 280, then find the other number.

Q.64. If least prime factor of p is 3 and least prime factor of q is 5, then what is least

prime factor of (p+q)?

Q.65. On a morning walk, three persons step out together and their steps measure 30cm,36cm and 40cm respectively. What is the minimum distance each should walk so that each can cover the same in complete steps?

Q.66. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.

Q.67. The traffic lights at at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 am, at what time will they change simultaneously again?

Q.68. Determine the greatest 3-digit number which is exactly divisible by 8, 10 and 12.

Q.69. Three farmers have 490kg, 588kg and 882kg of wheat respectively. Find the maximum capacity of a bag so that the wheat can be packed in exact number of bags.

Q.70. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively.Find the maximum capacity of a container that can measure the diesel of the three containers exact number of times.