INFINITY

THINK BEYOND............

AN EDUCATIONAL INSTITUTE

XI MATHEMATICS

By Deepika Bhati

VOLUME 1

Unit - I Sets & Functions

Unit - II Algebra

Chapter 01

BASIC CONCEPTS OF SET THEORY

IMPORTANT TERMS & DEFINITIONS

The theory of sets is extremely vital and fundamental part of modern day Mathematics. The concept of sets is widely used in the foundation of relations, functions, logic, sequences, theory of probability etc. In day to day life, we very often use the word *collections* or *aggregates of objects of some kinds*. The theory of sets was developed by a German Mathematician Georg Cantor (1845–1918 AD). The word *'set'* was first of all used by him. Cantor used the word *'set'* for a collection of objects of any kind. According to him, *"A set is any collection into a whole of definite and distinct objects of our intuition or thought."* By the objects being 'distinct', Cantor meant that *no object should be repeated in the collection* and by the objects being 'definite', he meant that *given an object we must be able to decide whether that object belongs to the collection or not*. By the term 'collection into a whole', he meant that *the objects should have certain properties*. But Cantor's definition of set was not acceptable to some mathematicians as there may be collections of objects such that objects of the collection do not satisfy some common properties.

I want to tell you that this topic is a bit tricky! So you have to be highly alert and give it more attention than you can offer. (How do we do so, well, even I am not aware of this!)

01. Set: A set is *well-defined* collection of objects of any kind of our intuition or thought which are distinct and distinguishable. By the word 'distinct' we mean that ***no object should be repeated*** and by the word distinguishable, we mean that ***the object of the collection must be known*** *i.e.,* given any object ***we must be able to decide whether that object belongs to the collection or not***. The objects of a set are taken as distinct only on the ground of simplicity. Generally *objects of a set have a common property* and the objects outside this collection *do not* have this property. The objects of the collection are called the ***elements or members of the set***.
e.g. Let A={*a, e, i, o, u*}. Here the elements of A are distinguishable as well as distinct. Hence A is a set.
Let B={*x*: *x* is an intelligent person of Delhi}. Here elements are not distinguishable because if we select any person of Delhi, we can't say with the certainty whether he belongs to B or not, as there is no standard scale for evaluation of intelligence. Hence B is not a set.

❖ A set is represented by listing all its elements between the braces { } and by separating them from each other by commas (if there are more than one element).
❖ Sets are denoted by capital letters of English alphabet *viz.* A, B, C, S, U, X, Y, Z etc., while the elements are in general denoted by small letters *viz. a, b, x, y* etc.
❖ If '*x*' is an element of a set A then we write $x \in A$ (read as *x* belongs to A). Also if '*y*' is not an element of set A then we write $y \notin A$ (read as *y* does not belong to A). The **symbol ∈ is called the membership relation**.

02. Methods of representing a set:
a) Tabular form or Roster form: In this method of describing a set, all the elements of a set are listed separated by commas and are enclosed within braces { }.
e.g. The set of all positive odd integers lesser than 10 can be described by {1, 3, 5, 7, 9}.

❖ *Note that in roster form of a set an element is not generally repeated. Also the order in which the elements of a set are written is immaterial. Thus the set {1, 3, 5, 7, 9} and {3, 1, 9, 5, 7} are same.*
In fact, Repetition of elements and order of elements in roster form is immaterial.

b) Set Builder form or Rule Method: In set builder form, all the elements of a set posses a single common property. A variable x which stands for each element of the set is written inside braces and then after giving a colon ":" or oblique line "/", the property or properties $p(x)$, possessed by each element of the set is written within the braces itself. In this description the braces stand for ***"the set of all"*** and the colon stands for ***"such that"***.

e.g. The set A= {1, 3, 5} is written as A= $\{x$: $x \in$N, x is an odd number and $x \le 5\}$ in the set builder form. It is read as "the set of all x such that x is a positive odd number less than or equal to 5".

03. The Empty set (or Null set or Void set): A set which does not contain any element is called the empty set and it is denoted by ϕ.

e.g. Let $\mathrm{A} = \{x : x$ is an even prime number greater than $2\}$. Then A is an empty set.

❖ *Note that the set {0} is not an empty set as it contains one element 0.*

04. Singleton set: A set having single element is called a singleton set. It is represented by writing down the element within the braces.

e.g. Let A={The set of present prime minister of India}. Then A is a singleton set.

05. Finite and Infinite set: A set which consists of a finite (definite) number of elements is called finite set, otherwise the set is called infinite set.

e.g. Let $\mathrm{A} = \{1, 2, 3, 4\}$ is a finite set and $\mathrm{B} = \{1, 2, 3, ...\}$ is an infinite set.

❖ *Note that an empty set is a finite set as it has no element!*

◆ ***Cardinal number of a finite set:*** *The number of elements in a finite set A is called the cardinal number of set A and is denoted by n(A). e.g. Let A= {a, e, i, o, u} then, n(A)= 5.*

06. Equal and Equivalent sets: Two sets A and B are said to be equal if they have exactly the same elements and we write A = B, i.e., sets A and B are equal if each element of A is an element of B and each element of B is an element of A. Otherwise the sets are said to be unequal and we write $\mathrm{A} \ne \mathrm{B}$. Also note that the order in which the elements in the two sets have been written down is immaterial.

e.g. Let $\mathrm{A} = \{a, b, c, d, e\}$ and $\mathrm{B} = \{c, d, a, b, e\}$, then $\mathrm{A} = \mathrm{B}$.

❖ *Note that two sets A and B are equal if* $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.

Two finite sets A and B are said to be equivalent if they have the same number of elements *i.e.* ***same cardinal number***. Thus sets A and B are equivalent iff $n(\mathrm{A}) = n(\mathrm{B})$ and, we write $\mathrm{A} \approx \mathrm{B}$.

e.g. Let $\mathrm{A} = \{a, e, i, o, u\}$ and $\mathrm{B} = \{1, 2, 3, 4, 5\}$, then $n(\mathrm{A}) = n(\mathrm{B}) = 5$. Therefore, $\mathrm{A} \approx \mathrm{B}$.

❖ *Note that the equal sets are equivalent but equivalent sets may or may not be equal.*

07. Subsets, Supersets and Proper subsets: If every element of a set A is also an element of a set B, then **A is called a subset of B** (or **A is contained in B**) and we write $\mathbf{A} \subseteq \mathbf{B}$. If there exist at least one element of A which does not belong to B, then A is not a subset of B and we write $\mathbf{A} \not\subset \mathbf{B}$.

Thus $\mathrm{A} \subseteq \mathrm{B} \Leftrightarrow \{x \in \mathrm{A} \Rightarrow x \in \mathrm{B}\}$.

e.g. Let $\mathrm{A} = \{1, 2, 3\}$ and $\mathrm{B} = \{1, 2, 3, 4, 5\}$. Then $\mathrm{A} \subseteq \mathrm{B}$.

<u>Superset of a set:</u> The statement $\mathrm{A} \subseteq \mathrm{B}$ can also be expressed equivalently by writing $\mathrm{B} \supseteq \mathrm{A}$ (read as '**B is a superset of A**'). Also a set A is said to be a superset of set B, if B is a subset of A *i.e.,* each element of B is an element of A. If A is a superset of B, we write $\mathrm{A} \supseteq \mathrm{B}$.

e.g. Let $\mathrm{A} = \{1, 2, 3, 4, 5\}$ and $\mathrm{B} = \{2, 4, 5\}$. Here B is a subset of A, therefore, A is a superset of B.

<u>Proper subset of a set:</u> A set A is said to be a proper subset of a set B if A is a subset of B and $\mathrm{A} \ne \mathrm{B}$ *i.e.,* if

a) every element of A is an element of B and

b) B has at least one element which is not an element of set A.

Thus $A \subset B$ or $B \supset A$ (read as '**A is a proper subset of B**'). Also if A is not a proper subset of B, then we write $A \not\subset B$.

e.g. Let $A = \{1,2,3\}$ and $B = \{1,2,3,4,5\}$. Then $A \subset B$. Also let $A = \{1,2,3\}$ and $B = \{2,3,1\}$. Then $A \not\subset B$ as $A = B$.

◆ Note the followings :

(a) If $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$.

(b) As the empty set ϕ has no elements, so we can say that ϕ is a subset of every set.

(c) Every set A is a subset of itself, i.e., $A \subset A$.

(d) If A and B are two sets such that $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

08. Intervals as subset of real numbers: Consider $a, b, x \in \mathrm{R}$ and $a < b$. Here R represents the set of real numbers.

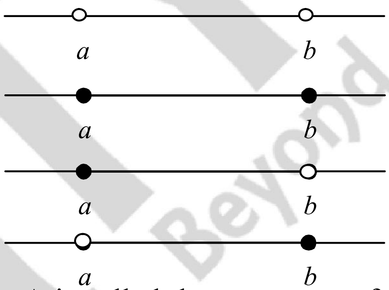
a) Open interval: (a,b) or $]a,b[$ or $a < x < b$

b) Closed interval: $[a,b]$ or $a \le x \le b$

c) Semi-open and semi-closed intervals:

i) $[a,b)$ or $[a,b[$ or $a \le x < b$

ii) $(a,b]$ or $]a,b]$ or $a < x \le b$.



09. Power set: The set or family of all the subsets of a given set A is called the power set of A and is denoted by P(A). Note that in P(A), every element is a set.

Thus symbolically, $\mathrm{P(A)} = \{\mathrm{X} : \mathrm{X} \subseteq \mathrm{A}\}$. Hence $\mathrm{X} \in \mathrm{P(A)} \Leftrightarrow \mathrm{X} \subseteq \mathrm{A}$.

Also, $\phi \in \mathrm{P(A)}$ and $\mathrm{A} \in \mathrm{P(A)}$ for all set A.

e.g. Let $A = \{1,2\}$ then $\mathrm{P(A)} = \{\phi, \{1\}, \{2\}, \{1,2\}\}$.

❖ *Note that as the set A with m number of elements has* 2^m *subsets so, P(A) has* 2^m *elements i.e.,* $n[\mathrm{P(A)}] = 2^m$, *where* $m = n(A)$.

10. Universal set: In any discussion in set theory we need a set such that all the sets under consideration in that discussion are its subsets. Such a set is called the universal set for that discussion. In other words, any set which is superset of all the sets under consideration is called the universal set and is **denoted by S or U**. A universal set can be chosen arbitrarily for any discussion of given sets, but once chosen, it is fixed for that discussion of the sets.

e.g. Let A={1, 2, 3}, B={3, 4, 6, 9} and C={0, 1}. In this case we can take U={0, 1, 2, 3, 4, 5, 6, 7, 8, 9} as the universal set.

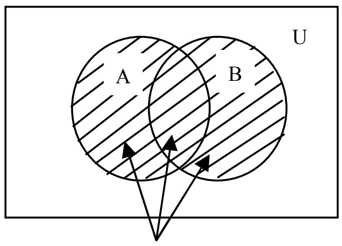
11. Operation on sets:

a) Union of sets: The union of two sets A and B is the set of all those elements which are either in set A or B or in both. This set is denoted by $A \cup B$. Thus $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Clearly $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$.

e.g. Let A={0, 1} and B={2, 3} then $A \cup B$={0, 1, 2, 3}.

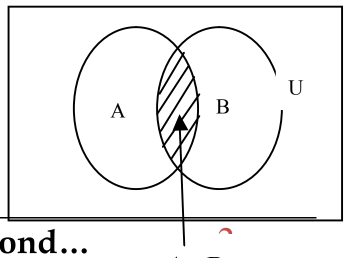
❖ *Note that if* $A \subseteq B$ then $A \cup B = B$.



A∪B

b) Intersection of sets: The intersection of set A and B is the set of all the elements which are common to both A and B. This set is denoted by $A \cap B$. Thus $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Clearly $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$.



A∩B

◆ Note the followings:

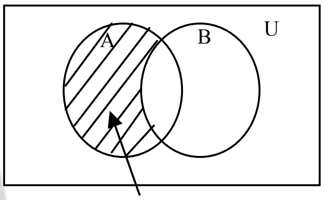
a) *If* $A \subseteq B$ *then* $A \cap B = A$ *and if* $B \subseteq A$ *then* $A \cap B = B$.

b) $(A \cap B) \cup A = A$ *and* $(A \cap B) \cup B = B$.

c) $(A \cup B) \cap A = A$ *and* $(A \cup B) \cap B = B$.

d) *If* $A \cap B = \phi$ *then, A and B are **disjoint sets**.*

c) Difference of sets: The difference of set A and B, in this order, is the set of all those elements of A which are not the elements of B. It is denoted by (A – B).



A–B

Thus $\mathrm{A} - \mathrm{B} = \{x : x \in \mathrm{A} \text{ and } x \notin \mathrm{B}\}$.

Clearly, $x \in \mathrm{A} - \mathrm{B} \Leftrightarrow x \in \mathrm{A} \text{ and } x \notin \mathrm{B}$.

❖ *Note that* $A - B = A \cap B'$ *i.e.* $A - B = A \cap \overline{B}$.

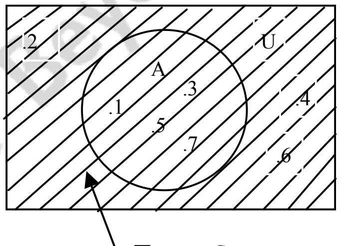
Venn diagram is the pictorial representation of sets in which a set is represented by the region within a closed curve, usually circle or ellipse, inside the universal set. The universal set U is represented by a rectangular region. An element of a set *A* is represented by a point within the circle which represents *A*.

d) Complement of a set:

Let U be the universal set and *A* is a subset of U. Then the complement of A (denoted by A' or A^{C} or $\overline{\mathrm{A}}$) with respect to U is the set of all those elements of U which are not the elements of A.

Thus $\mathrm{A}' = \{x : x \in \mathrm{U} \text{ and } x \notin \mathrm{A}\}$.

e.g. Let $\mathrm{U} = \{1, 2, 3, 4, 5, 6, 7\}$ and $\mathrm{A} = \{1, 3, 5, 7\}$ then, $\mathrm{A}' = \{2, 4, 6\}$.



A' or $\overline{\mathrm{A}}$ or A^{C}

◆ Note the followings:

(i) $\phi' = \mathrm{U}$ **(ii)** $\mathrm{U}' = \phi$

(iii) $(\mathrm{A}')' = \mathrm{A}$ **(iv)** $\mathrm{A}' = \mathrm{U} - \mathrm{A}$.

Disjoint sets: *Two sets A and B are disjoint if* $A \cap B = \phi$. *That is, for disjoint sets A and B, there won't be any element which is present in both (i.e., there won't be any common element).*

☞ **Note :** If $\mathrm{A} \cap \mathrm{B} \neq \phi$ then, A and B are said to be **overlapping sets** or **intersecting sets** or **not disjoint sets**.

12. Relations between sets for the application based problems: If sets are not disjoint, then we have following relations,

(a) $n(\mathrm{A \text{ or } B}) = n(\mathrm{A} \cup \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$

(b) $n(\mathrm{A \text{ and } B}) = n(\mathrm{A} \cap \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cup \mathrm{B})$

(c) $n(\mathrm{A \text{ or } B \text{ or } C}) = n(\mathrm{A} \cup \mathrm{B} \cup \mathrm{C}) = n(\mathrm{A}) + n(\mathrm{B}) + n(\mathrm{C}) - n(\mathrm{A} \cap \mathrm{B}) - n(\mathrm{B} \cap \mathrm{C}) - n(\mathrm{C} \cap \mathrm{A}) + n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$

13. Some Important Laws:

(a) Commutative laws

$$\mathrm{A} \cup \mathrm{B} = \mathrm{B} \cup \mathrm{A},\ \mathrm{A} \cap \mathrm{B} = \mathrm{B} \cap \mathrm{A}$$

(b) Associative laws

$$(\mathrm{A} \cup \mathrm{B}) \cup \mathrm{C} = \mathrm{A} \cup (\mathrm{B} \cup \mathrm{C}),\ (\mathrm{A} \cap \mathrm{B}) \cap \mathrm{C} = \mathrm{A} \cap (\mathrm{B} \cap \mathrm{C})$$

(c) Distributive laws

$$\mathrm{A} \cap (\mathrm{B} \cup \mathrm{C}) = (\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{C}),\ \mathrm{A} \cup (\mathrm{B} \cap \mathrm{C}) = (\mathrm{A} \cup \mathrm{B}) \cap (\mathrm{A} \cup \mathrm{C})$$

For sets A and B, $(A-B)\cup(B-A)$ is called **symmetric difference** of sets A and B. It is denoted by $A \Delta B$.

(d) De Morgan's laws

$(A\cup B)' = A'\cap B'$, $(A\cap B)' = A'\cup B'$

(e) Idempotent laws

$A\cup A = A$, $A\cap A = A$

(f) Complement laws

$A\cup A' = U$, $A\cap A' = \phi$

(g) Identity laws

$A\cup\phi = A$, $A\cap U = A$, $A\cap\phi = \phi$, $A\cup U = U$

(h) Double complementation law (Involution law)

$(A')' = A$

(i) Laws of empty set and universal set

$\phi' = U$, $U' = \phi$.

14. Symbols and their meanings:

S. No.	**Symbol**	**Meaning**
01.	N	Set of natural numbers
02.	I *or* Z	Set of integers
03.	Q	Set of rational numbers
04.	T	Set of irrational numbers
05.	R	Set of real numbers
06.	C	Set of complex numbers
07.	$\in$	is an element of (or belongs to)
08.	$\notin$	is not an element of (or does not belong to)
09.	S *or* ξ *or* U	Universal set
10.	: *or* /	Such that
11.	ϕ	Empty set or Null set
12.	$\subseteq$	is subset of
13.	$\supseteq$	is superset of
14.	$\subset$	is proper subset of
15.	$\supset$	is proper superset of
16.	$\cup$	Union
17.	$\cap$	Intersection
18.	$\forall$	For all
19.	$\Rightarrow$	Implies
20.	$\Leftarrow$	is implied by
21.	$\Leftrightarrow$	Implies that and implied by
22.	ξ	Zai (Greek Letter)